When, Where, and How Much to Invest for Enhancing Transportation Network Performance: Insights to Augment Decision Making

Presented by:
Sabya Mishra[a], Mihalis M. Golias[a], and Timothy Welch[b]
[a] University of Memphis
[b] Georgia Institute of Technology

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Motivation (1)

- State DOTs must allocate available budget to set of projects
- Divisions within DOT focus on specific MOE
  - Congestion
  - Consumer surplus
  - Safety
  - Air Quality
- Focus in one MOE may impact other

Motivation (2)

- Budget is often limited
- Empirical models does not work (lack of behavioral component)
- Need to consider user behavior
- Single year versus multi-year investment
- MOEs are often conflicting

Motivation (3)

- Key question remains

  "When, where and how much do we need to invest such that overall goal of a transportation agency is satisfied"

  "Would it be prudent to invest now or wait"

Background

- The problem is multi-objective
- Considers multiple players
  - Decision makers (leaders)
  - Users (followers)
- Stackelberg's game
- Problems are often non-linear and difficult to solve

Purpose of Project

- Produce a method of prioritizing projects
  - Consider Total System Travel Time (TSTT) performance measure
- Allocate the budget to priority links
- Provide State DOTs with a decision making tool

Methodology

- Bi-Level Optimization

- Planners
  - Upper Level Problem (ULP)
  - Minimize Total System Travel Time (TSTT)

- Users
  - Lower Level Problem (LLP)
  - Traffic Assignment (UE)

Data Required

- Number of Links in the network
  - Capacity
  - Length
  - Free Flow Travel Time
  - Alpha and Beta parameters
  - Connecting Nodes
- O/D Matrix
- Budget

Formulation

Upper Level problem (ULP)

Objective Function :

$$\textbf{Minimize TSTT} = \sum_{a} x_a\, t_a\,(x_a, y_a)$$

Subject to:

$$\sum_{\forall_a} g_a(y_a) \leq B$$

$$0 \leq\ y_a\ \leq c_a : \forall\, a \in A$$

Where:

$TSTT$: Total System Travel Time

x_a : Flow for link a

y_a : Capacity expansion for link ‘a’ (nonnegative real value)

t_a : Travel time for link a

$t_a(x_a, y_a)$: Travel cost on link a as a function of flow and capacity expansion

$g_a(y_a)$: improvement cost function for link ‘a’

B : Budget (nonnegative real value)

Formulation

Lower Level problem (LLP)

$$\text{Minimize TT} = \sum_{a \in A} \int_0^{x_a} t_a(x_a, y_a)\,dx$$

Subject to:

$$q_{ij} = \sum_{k \in k^{ij}} f_k^{ij} \ \forall (i,j) \in IJ$$

$$x_a = \sum_{(i,j) \in IJ} \sum_{k \in K^{ij}} \delta_{ak}^{ij} f_k^{ij}, \quad \forall_a \in A$$

$$f_k^{ij} \geq 0, \ \ \forall_k \in k^{ij}, \qquad \forall (i,j) \in IJ,$$

$$q_{ij} \geq 0, \quad \forall (i,j) \in IJ$$

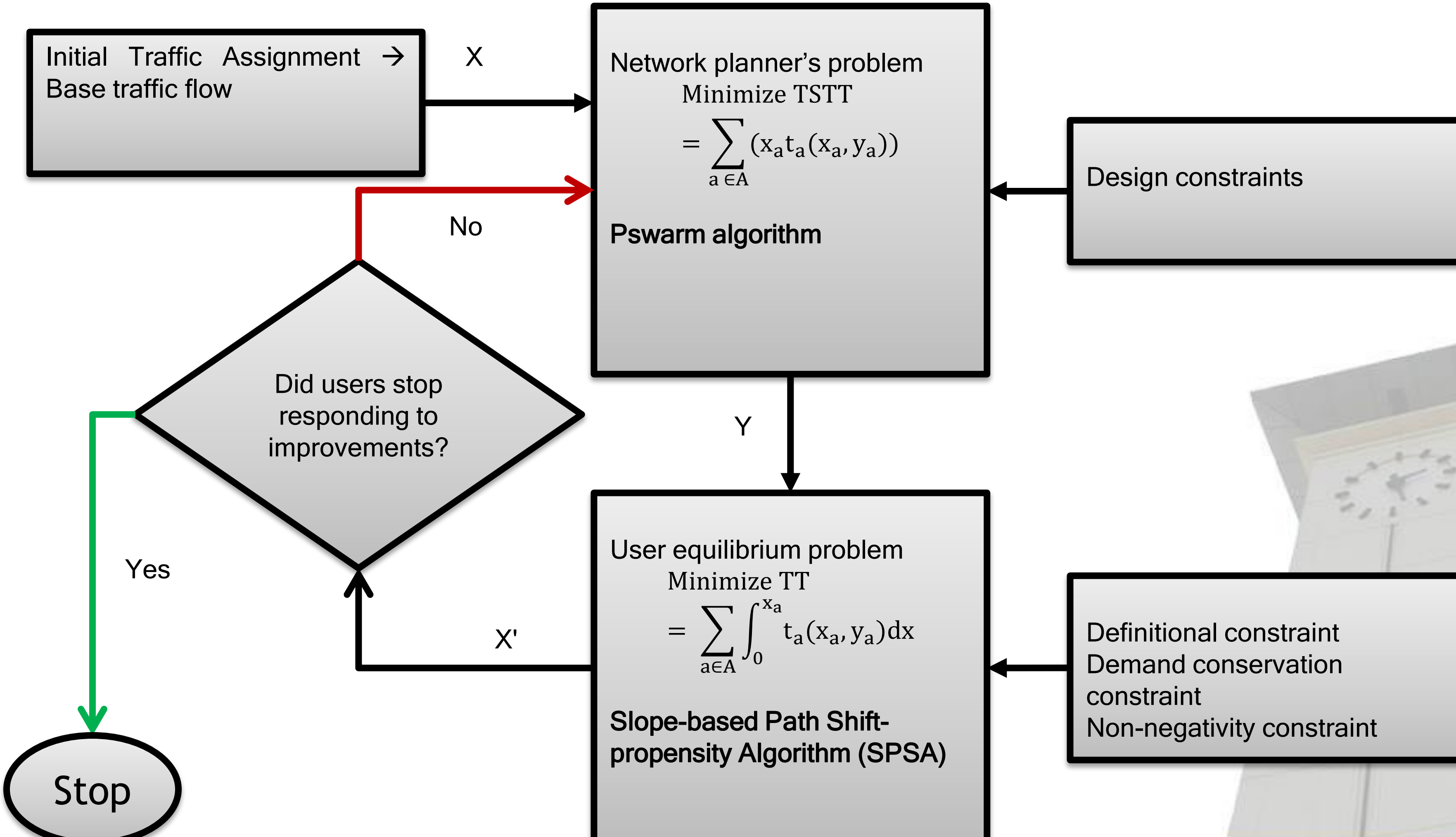
Notations

C_a	:	The capacity for link a
f_{ij}^{r}	:	Flow on path r, connecting each Origin-Destination (O-D) pair (i-j)
q_{ij}	:	Demand between each Origin-Destination (O-D) pair (i-j)
t_a	:	Travel time for link a
$t_a(x_a, y_a)$	:	Travel cost on link a as a function of flow and capacity expansion
x_a	:	Flow for link a
α_a	:	Constant, varying by facility type (BPR function)
β_a	:	Constant, varying by facility type (BPR function)
$\delta_{a,ij}^{r}$	:	binary variable 0,1 {1,if link a ∈A is on path k ∈ k^ij:0,otherwise}
t_o	:	Free flow time on link a
y_a	:	Capacity expansion for link ‘a’ (nonnegative real value)

Kth best algorithm for bi-level optimization

- Iteratively solves both upper and lower level problems
- Both upper and lower level can be solved using exact algorithms
- Often provides better solution than evolutionary algorithms (Karoonsoontawong & Waller 2006)
- Application:
  - Static traffic assignment
  - Dynamic traffic assignment
  - Other bi-level problem domains

Solution Approach



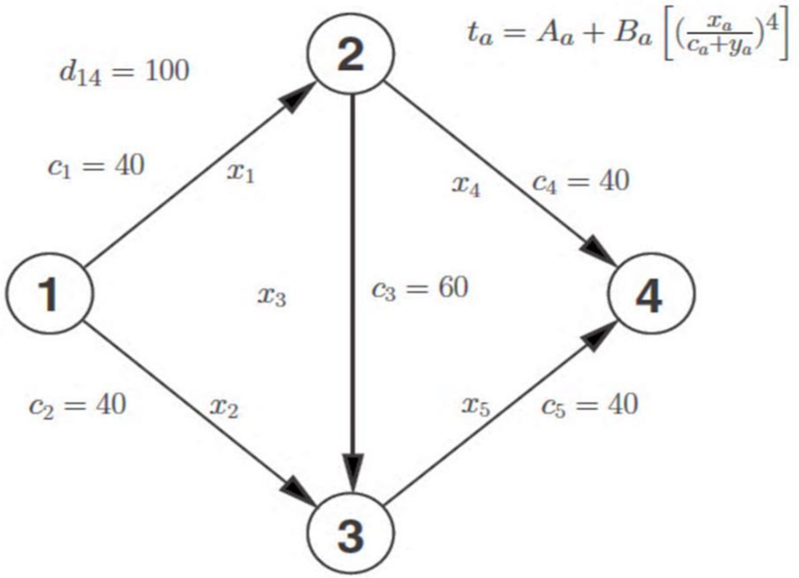
Slope-based Path Shift-propensity Algorithm (SPSA)

- Algorithm Characteristics:
  - Combines merits of simultaneous and sequential approach
    - Updates path sets for all O-D pair simultaneously
      - partially tackles the problem of order bias
    - Equilibrates one O-D pair at a time
      - leads to faster convergence
  - Incorporates behavioral realism in the flow update process
  - Convergence is theoretically proven
  - Simplicity of execution for easy deployment in practice
- Developed by Kumar and Peeta (2014)

Particle swarm (pswarm) algorithm

- Simulate behavior of particles that attempt to optimally explore some given solution space
- Population of particles is called **swarm**
- A particle flies in the solution space in search of optimal position
- Particle adjust its position and velocity using its own as well as other particles' experiences in the population
  - Combines the local search (own experience) with global search (population experience)
- Known to perform better than other global optimization methods such as genetic algorithm

Test Network 1



$$t_a(x_a, y_a) = A_a + B_a \left(\frac{x_a}{C_a + y_a} \right)^4$$

$$TSTT(y) = \sum_a (t_a(x_a, y_a).x_a + 1.5 d_a y_a^2$$

Arc a	A_a	B_a	C_a	d_a
1	4	0.60	40	2
2	6	0.90	40	2
3	2	0.30	60	1
4	5	0.75	40	2
5	3	0.45	40	2

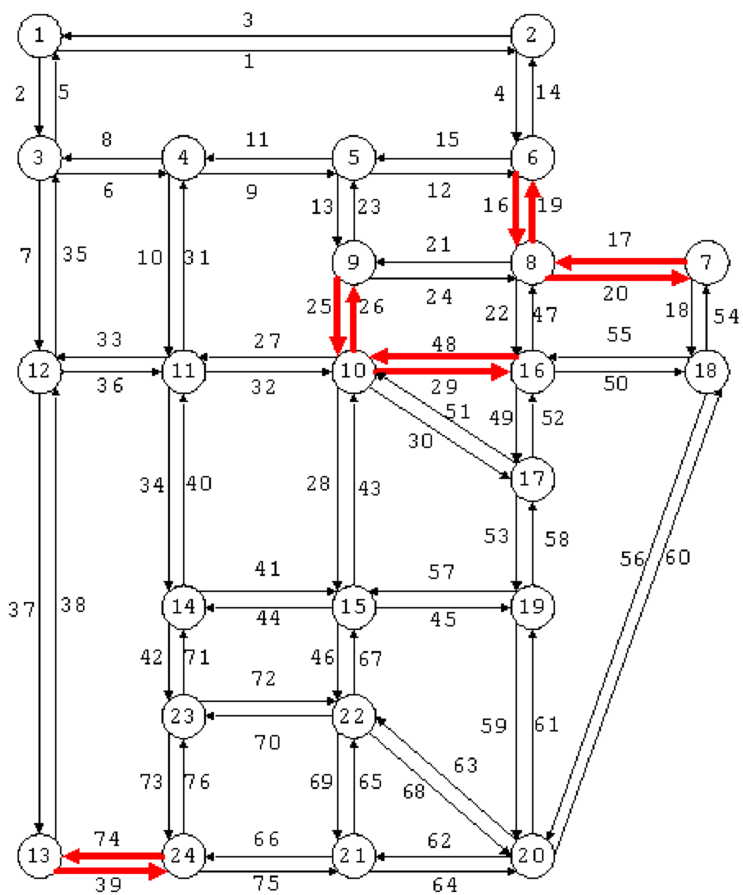
Comparison of Results

Case		MINOS	Hooke-Jeeves (H-J)	EDO	GA	Current Study
1	Demand =100					
	y_1	1.34	1.25	1.31	1.33	**1.33**
	y_2	1.21	1.20	1.19	1.22	**1.21**
	y_3	0.00	0.00	0.06	0.02	**0.00**
	y_4	0.97	0.95	0.94	0.96	**0.96**
	y_5	1.10	1.10	1.06	1.10	**1.10**
	Z	1200.58	1200.61	1200.64	1200.58	**1200.58**
2	Demand =150					
	y_1	6.05	5.95	5.98	6.08	**6.06**
	y_2	5.47	5.64	5.52	5.51	**5.46**
	y_3	0.00	0.00	0.02	0.00	**0.00**
	y_4	4.64	4.60	4.61	4.65	**4.64**
	y_5	5.27	5.20	5.27	5.27	**5.27**
	Z	3156.21	3156.38	3156.24	3156.23	**3156.21**
3	Demand =200					
	y_1	12.98	13.00	12.86	13.04	**12.98**
	y_2	11.73	11.75	12.02	11.73	**11.73**
	y_3	0.00	0.00	0.02	0.01	**0.00**
	y_4	10.34	10.25	10.33	10.33	**10.34**
	y_5	11.74	11.75	11.77	11.78	**11.74**
	Z	7086.12	7086.21	7086.45	7086.16	**7086.11**
4	Demand =300					
	y_1	28.45	28.44	28.11	28.48	**28.47**
	y_2	25.73	25.75	26.03	25.82	**25.71**
	y_3	0.00	0.00	0.01	0.08	**0.00**
	y_4	23.40	23.44	23.39	23.39	**23.41**
	y_5	26.57	26.56	26.58	26.48	**26.55**
	Z	21209.90	21209.91	21210.54	21210.06	**21209.90**

	Names of heuristics	Sources
GA	Genetic Algorithm	Mathew (2009)
H-J	Hooke-Jeeves algorithm	Abdulaal and LeBlanc (1979)
EDO	Equilibrium Decomposed Optimization (Bolzano search)	Suwansirikul et al. (1987)
MINOS	Modular In-core Non linear System	Suwansirikul et al. (1987)

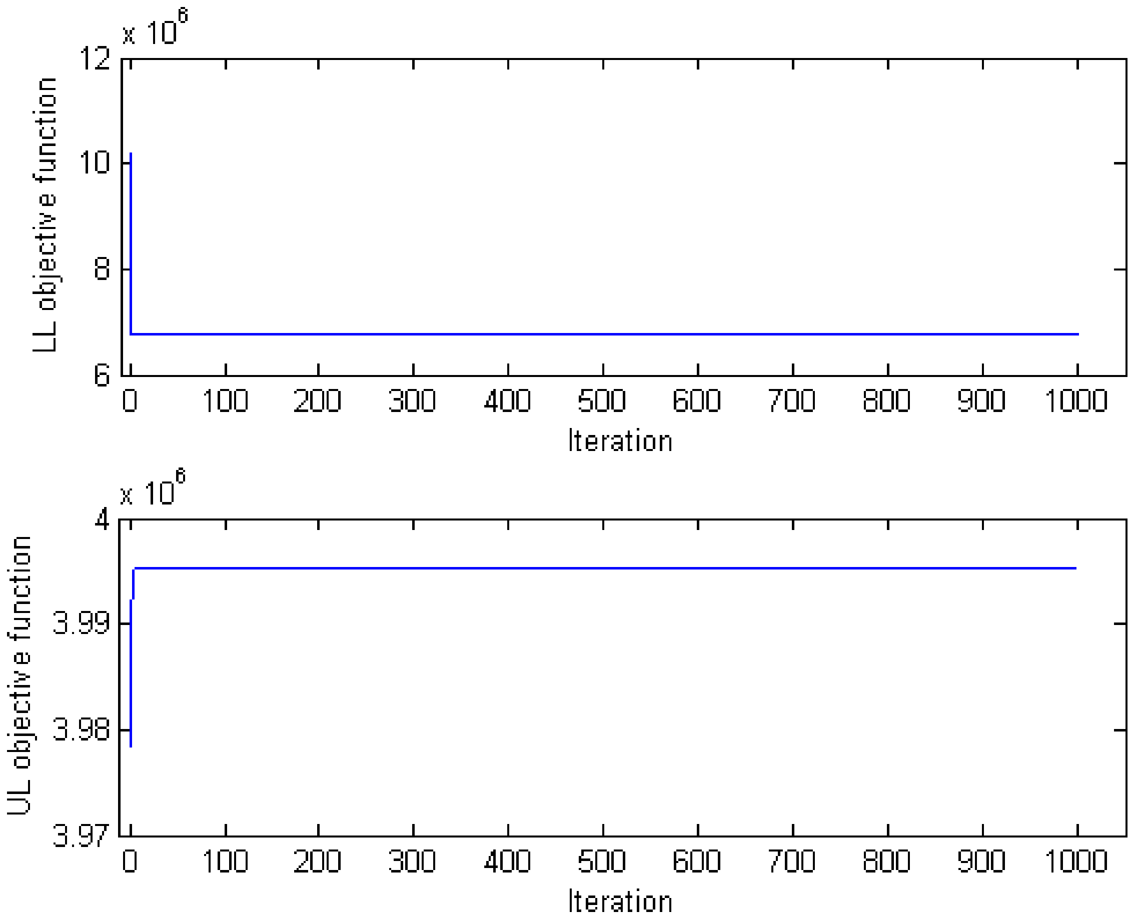
Test Network 2

candidate links are marked by red arrow

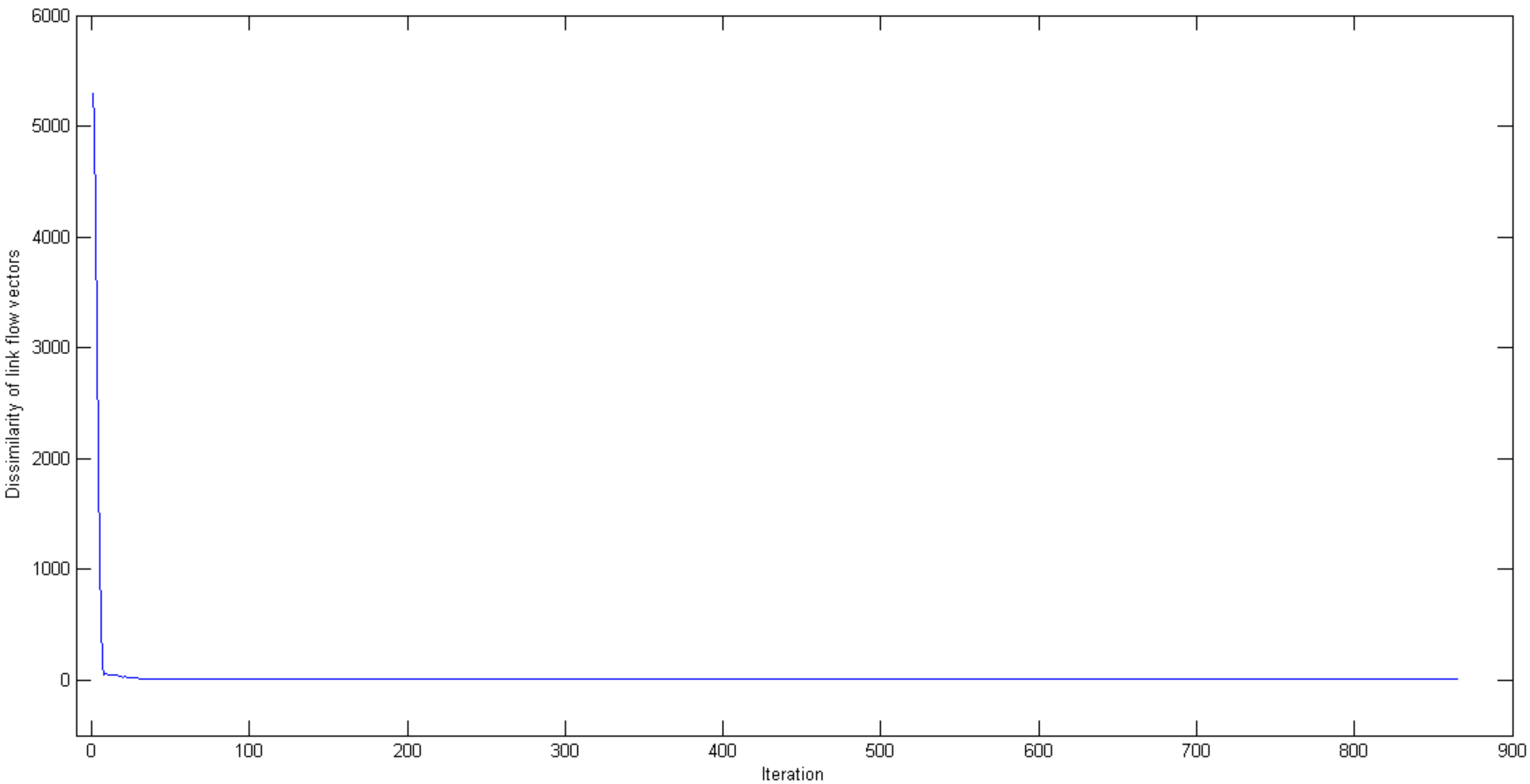


Sioux Falls Network

LL and UL objective functions



Dissimilarity of link flow vectors



Comparison of Results

Comparison of Results for Sioux Falls Network

Case	H-J	EDO	SA	SAB	GP	CG	QNew	PT	GA	Current Study
y16	4.8	4.59	5.38	5.74	4.87	4.77	5.3	5.02	5.17	**5.13**
y17	1.2	1.52	2.26	5.72	4.89	4.86	5.05	5.22	2.94	**1.35**
y19	4.8	5.45	5.5	4.96	1.87	3.07	2.44	1.83	4.72	**5.13**
y20	0.8	2.33	2.01	4.96	1.53	2.68	2.54	1.57	1.76	**1.32**
y25	2	1.27	2.64	5.51	2.72	2.84	3.93	2.79	2.39	**2.98**
y26	2.6	2.33	2.47	5.52	2.71	2.98	4.09	2.66	2.91	**2.98**
y29	4.8	0.41	4.54	5.8	6.25	5.68	4.35	6.19	2.92	**4.89**
y39	4.4	4.59	4.45	5.59	5.03	4.27	5.24	4.96	5.99	**4.45**
y48	4.8	2.71	4.21	5.84	3.76	4.4	4.77	4.07	3.63	**4.97**
y74	4.4	2.71	4.67	5.87	3.57	5.52	4.02	3.92	4.43	**4.4**
Zy	82.5	84.5	81.89	84.38	84.15	84.86	83.19	84.19	81.74	**80.99**

GP	Gradient Projection method
CG	Conjugate Gradient projection method
QNEW	Quasi-NEWton projection method
PT	PARTAN version of gradient projection method

Comparison of Results

Comparison of Results for Sioux Falls Network for different demand level

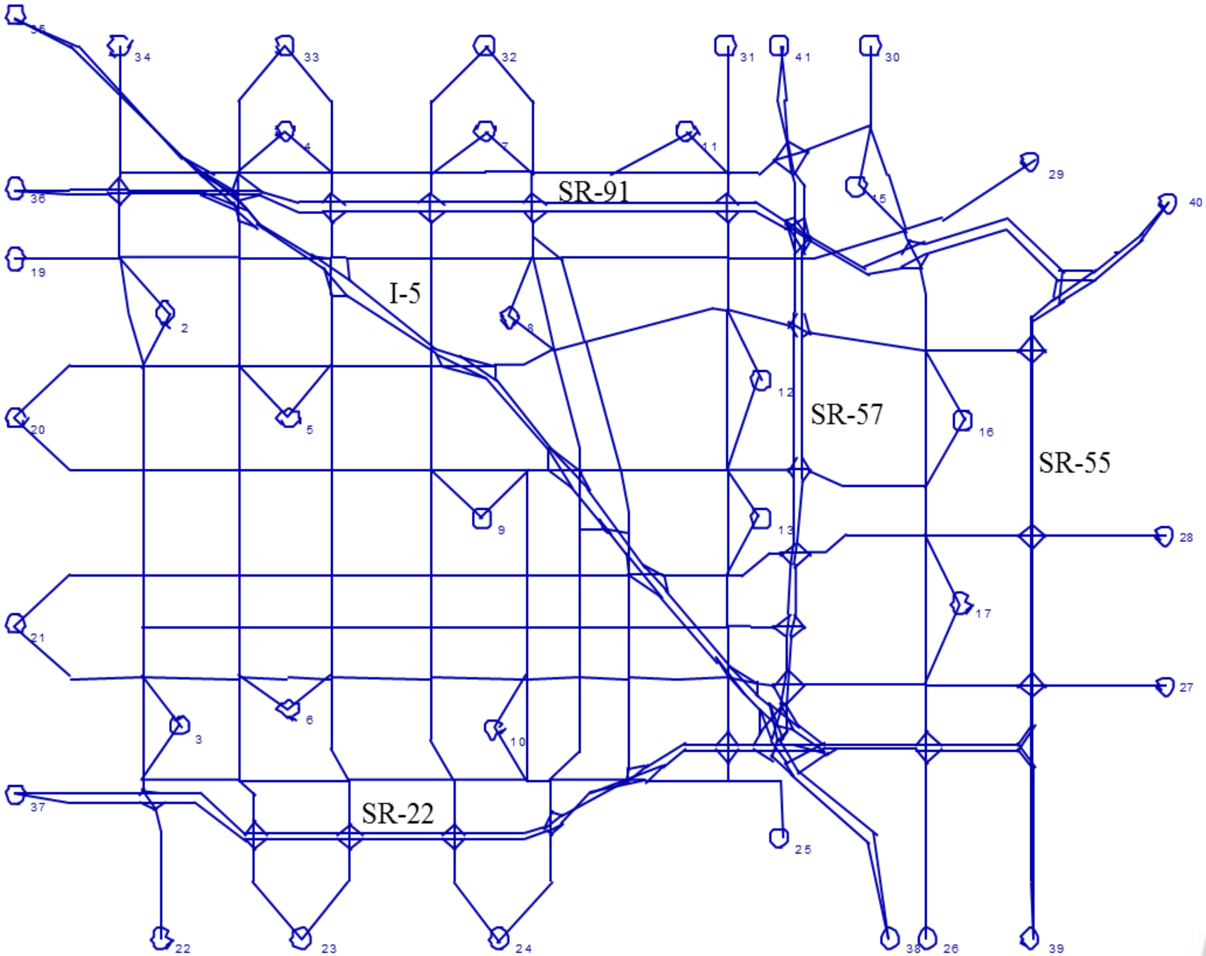
Demand Scenario	SAB	GP	CG	QNew	PT	EDO	IOA	GA	Current Study
0.8	51.76	48.38	48.78	48.84	48.81	49.51	53.58	48.92	**48.15**
Itr.	*14*	*10*	*3*	*4*	*9*	*7*	*28*	*59*	***29***
1	84.21	82.71	82.53	83.07	82.53	83.57	87.34	81.74	**80.99**
Itr.	*11*	*9*	*6*	*4*	*7*	*12*	*31*	*77*	***35***
1.2	144.86	141.53	141.04	141.62	142.27	149.39	150.99	137.92	**135.80**
Itr.	*9*	*11*	*10*	*7*	*9*	*12*	*31*	*67*	***36***
1.4	247.8	246.04	246.04	242.74	241.08	253.39	279.39	232.76	**229.22**
Itr.	*15*	*9*	*6*	*5*	*7*	*17*	*16*	*78*	***36***
1.6	452.01	433.64	408.45	409.04	431.11	427.56	475.08	390.54	**380.91**
Itr.	*14*	*9*	*9*	*9*	*11*	*19*	*40*	*83*	***40***

GA	Genetic Algorithm
GP	Gradient Projection method
CG	Conjugate Gradient projection method
QNEW	Quasi-NEWton projection method
PT	PARTAN version of gradient projection method

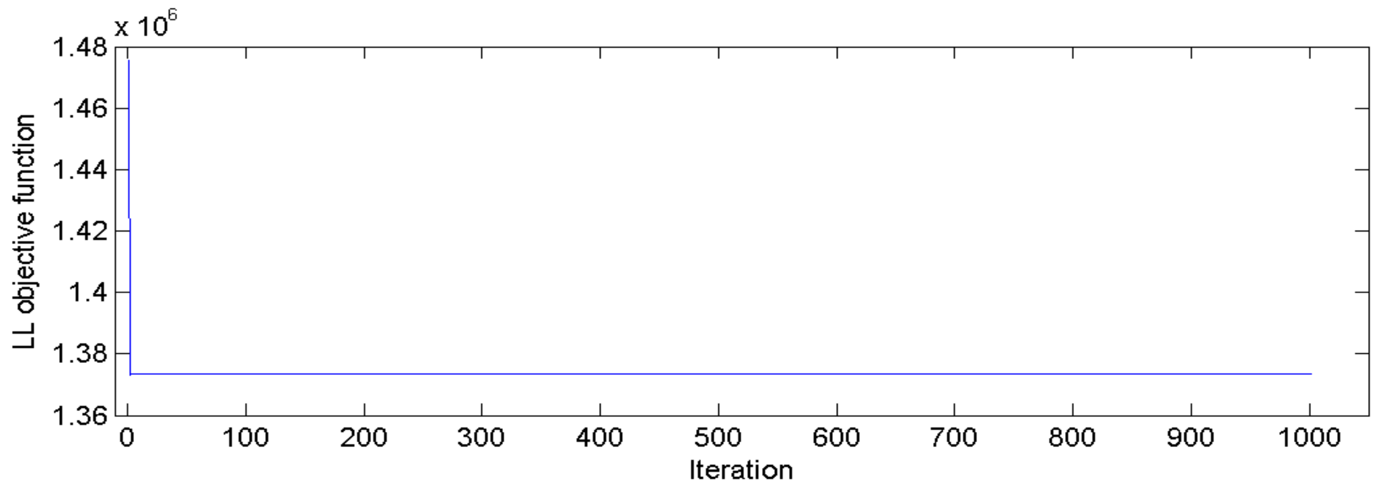
Application

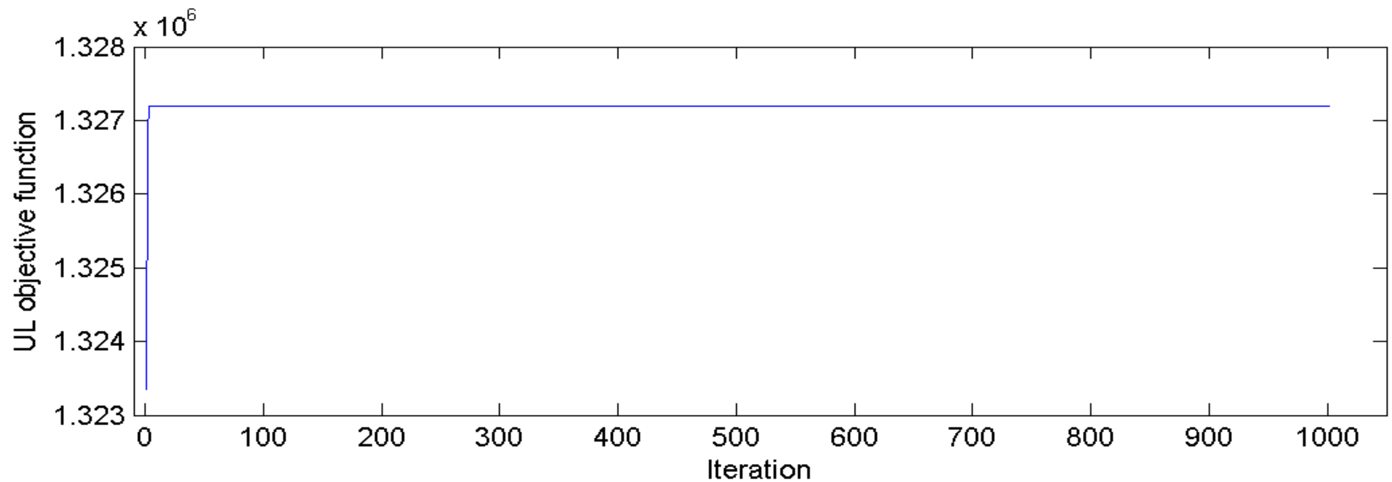
Network	Nodes	Links	Zones	O-D pairs with non-zero demand
Anaheim	416	914	38	1,416
Chicago Sketch	933	2,950	387	93,135
Washington DC	1,752	4,420	225	50,625
Atlanta	1,102	2,295	144	20,736

Anaheim

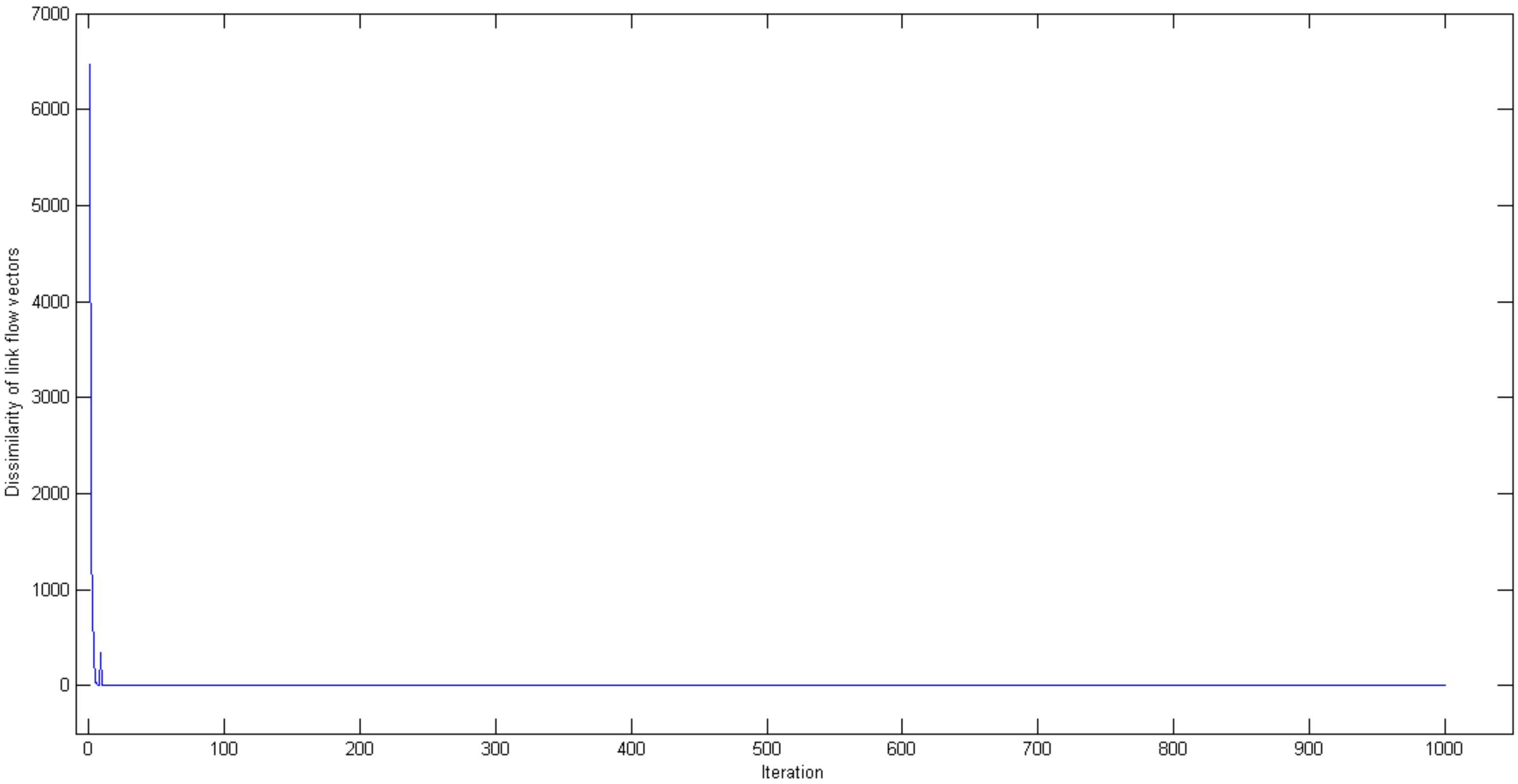


LL and UL objective functions

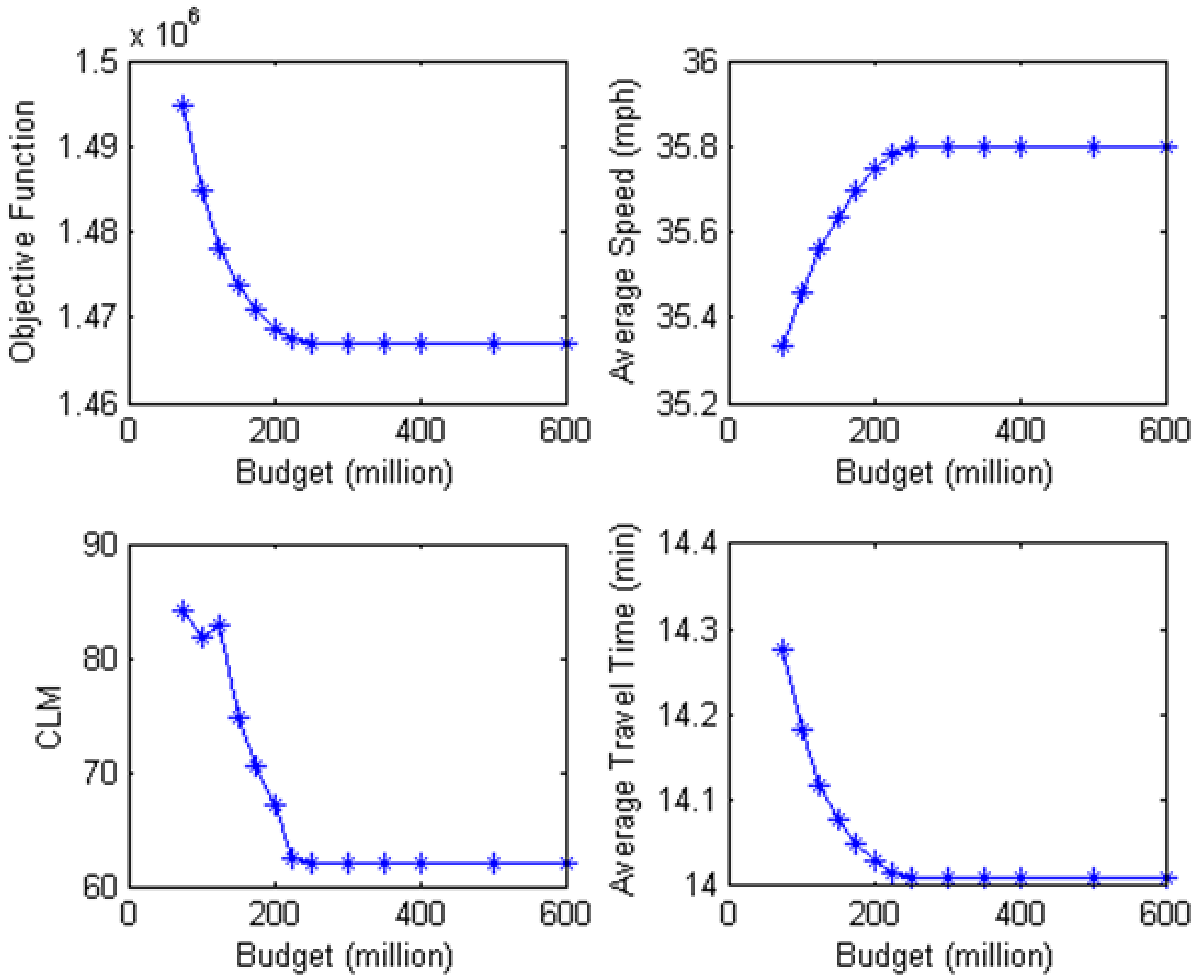




Dissimilarity of link flow vectors

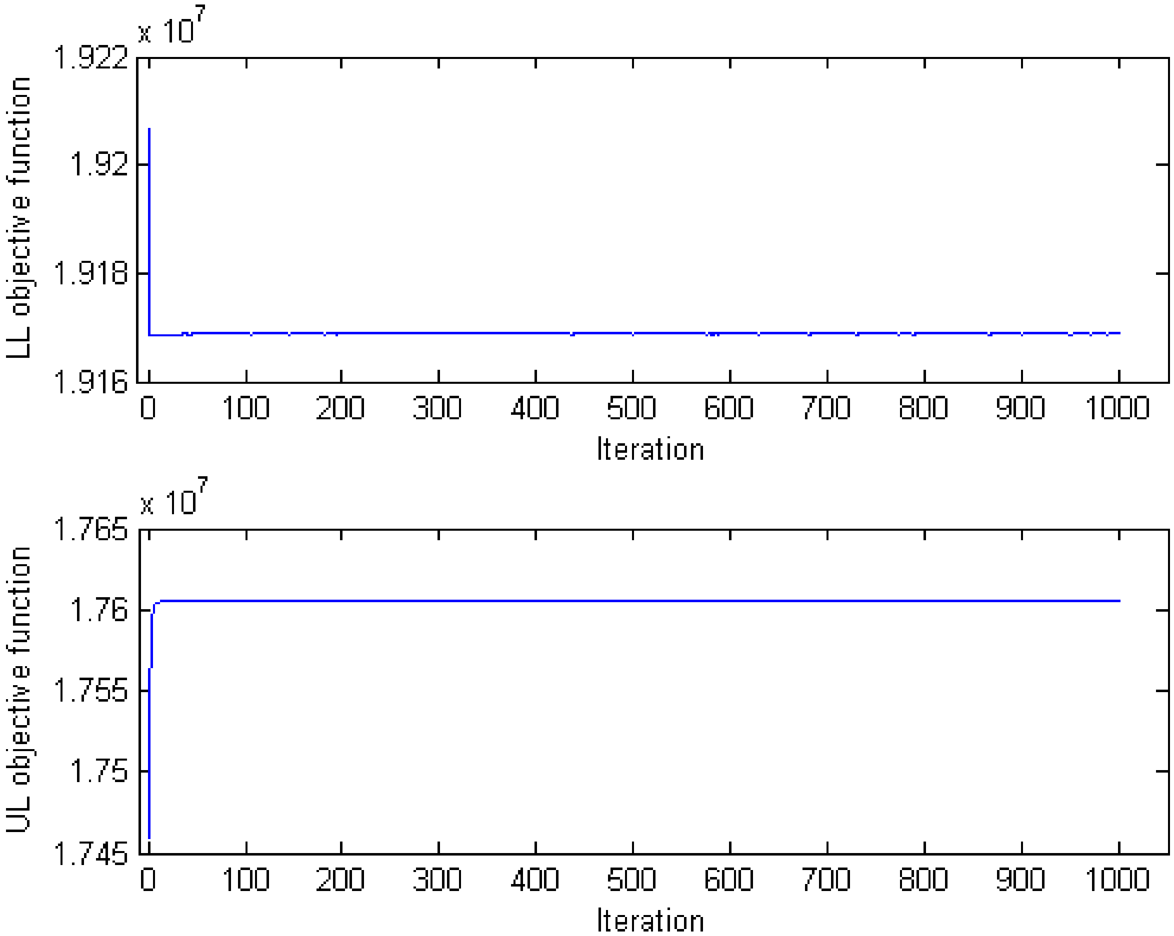


Anaheim Investment Scenarios



Chicago Sketch

LL and UL objective functions



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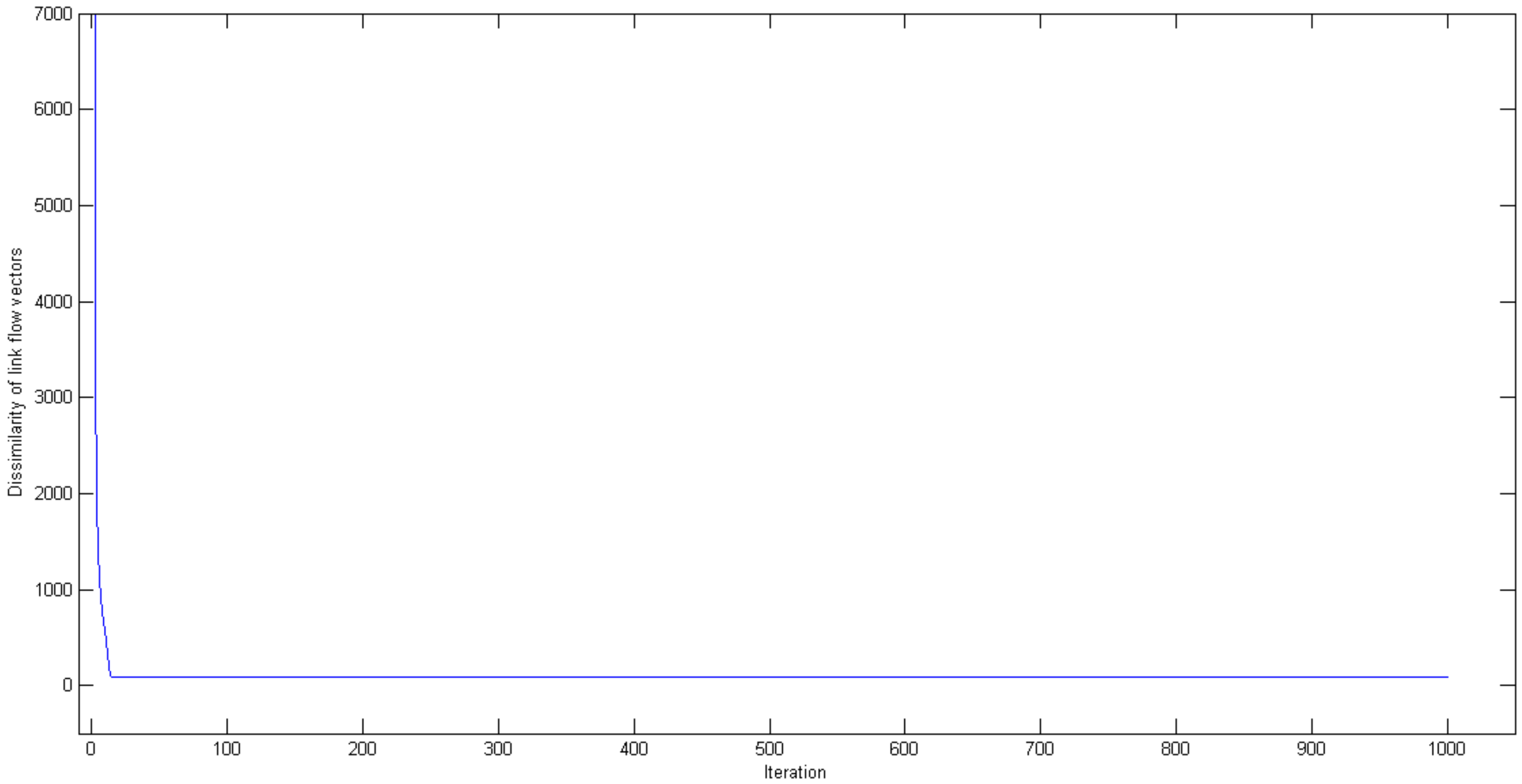
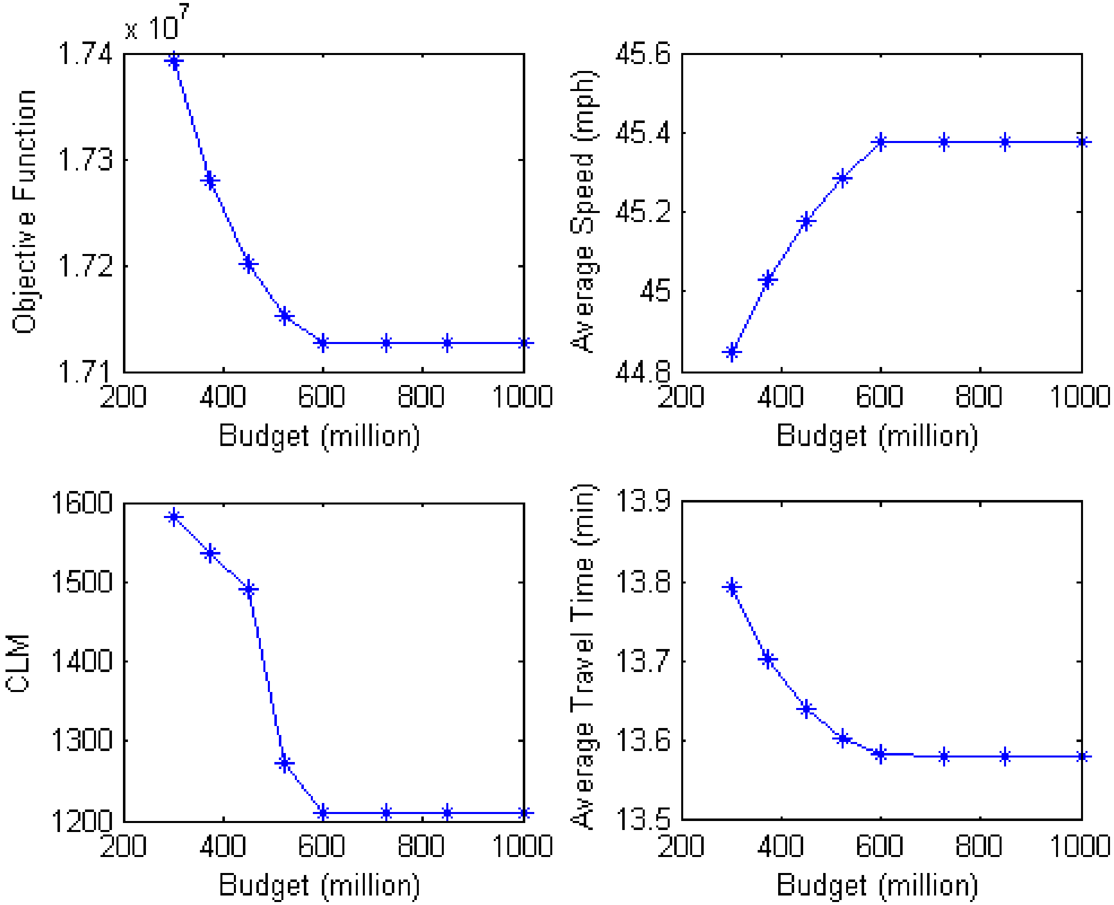


Chicago Sketch Investment Scenarios



Chicago Investment Scenarios

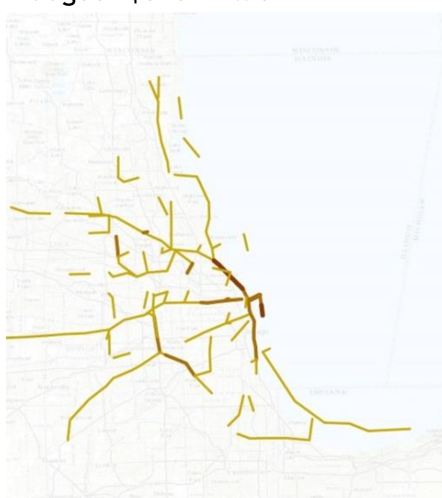
Chicago Sketch Network

Budget= $300 million

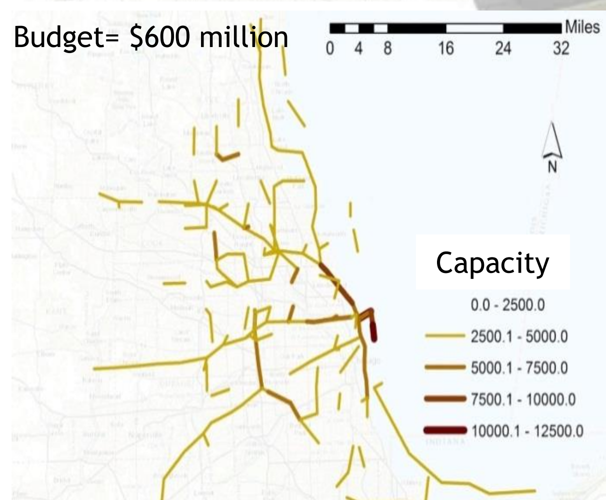
Budget= $375 million

Budget= $450 million

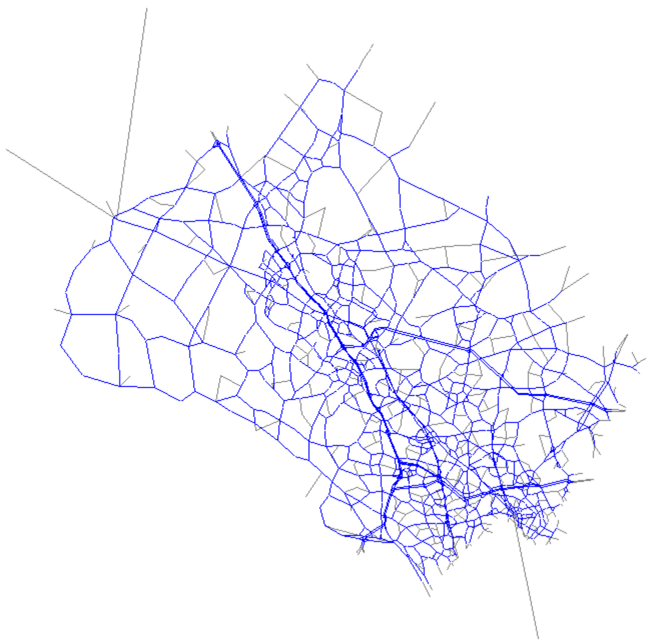
Budget= $525 million



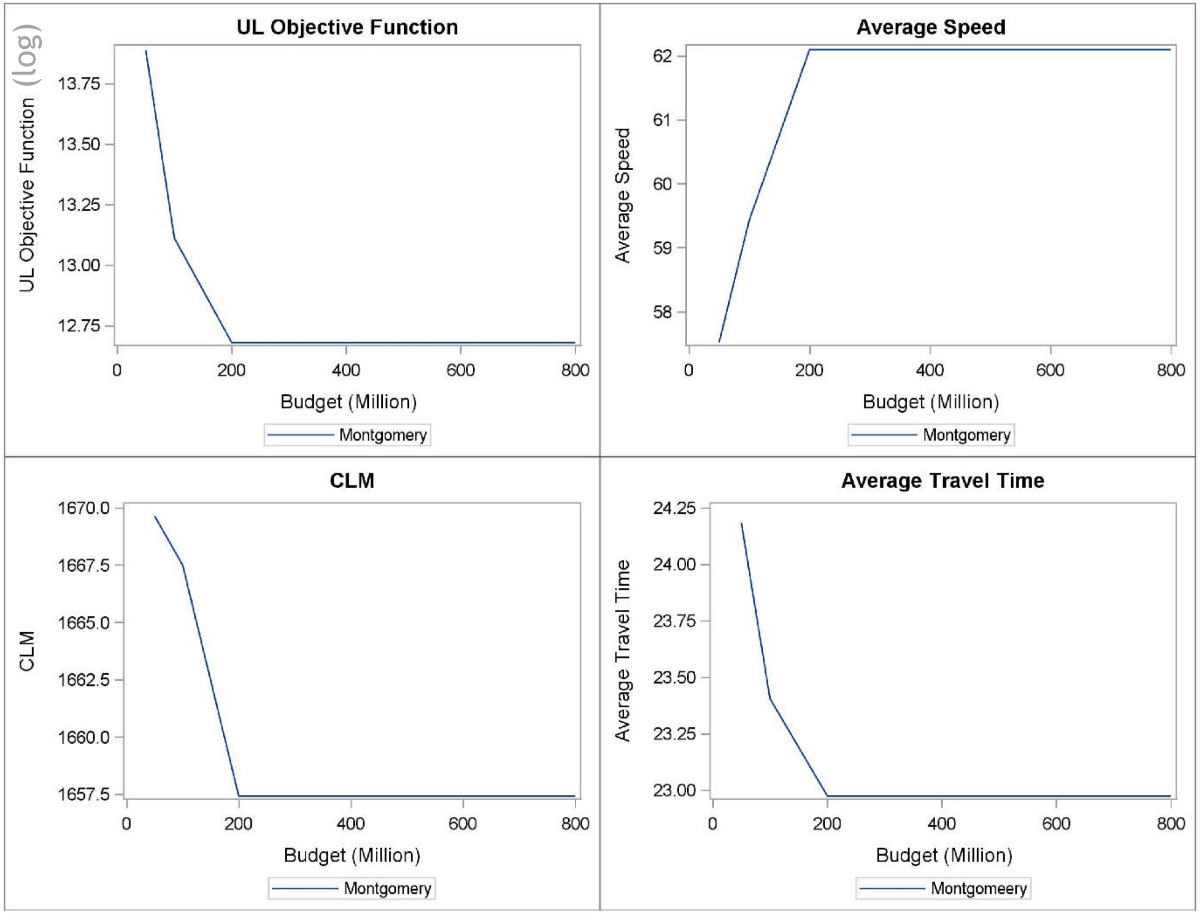
Budget= $600 million



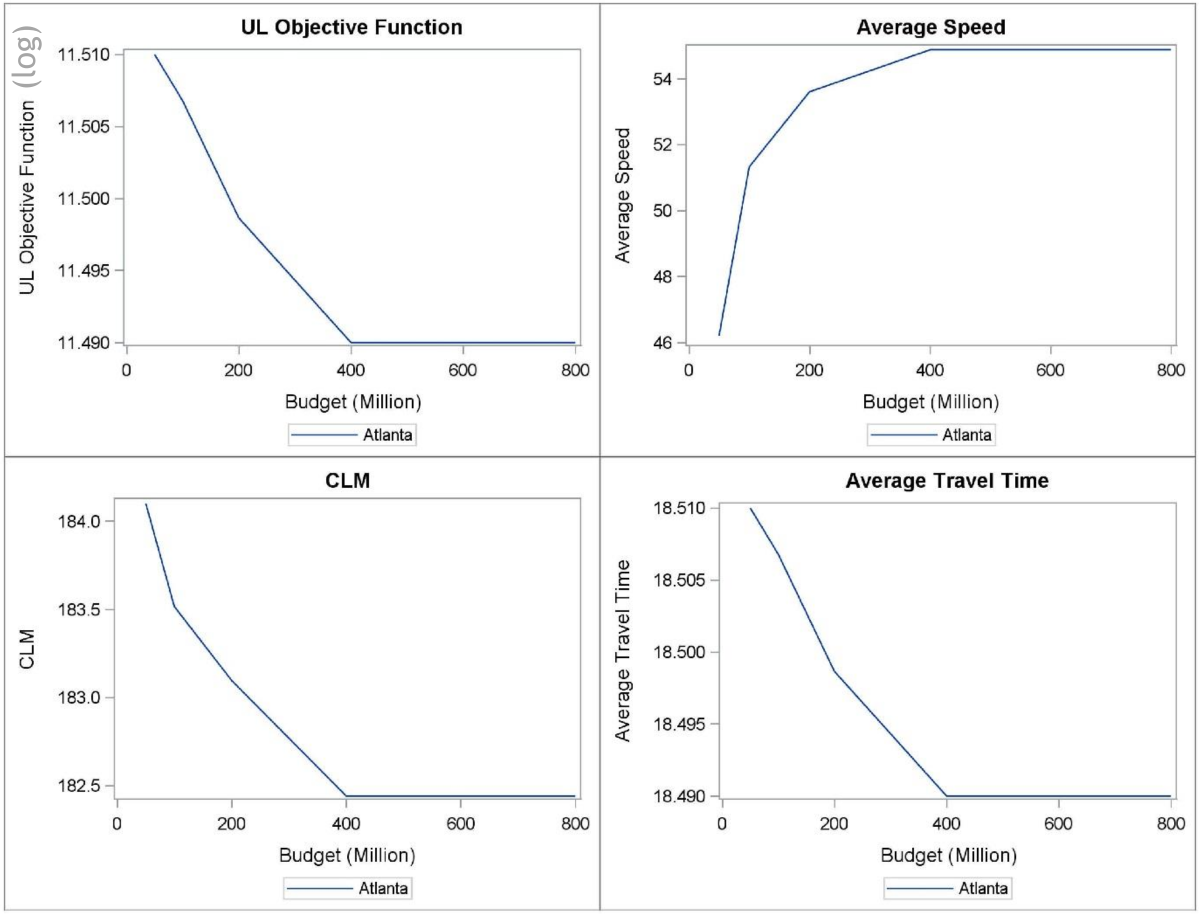
Montgomery County and Atlanta



Montgomery County Investment Scenarios



Atlanta Investment Scenarios



Summary and Conclusion

- In the investment decision making problem
  - One player (one upper level objective function)
  - One year analysis period
  - Kth best algorithm
  - Iterative approach between UL and LL
  - Compared to results from literature
  - Applied the procedure in real life networks

Limitations and Future Research

- Limitations
  - Kth best algorithm may not guarantee optimality
  - Computational time
- Future research
  - Converting single level
  - Considerations of multi year, multi objectives
  - Developing economic performance measures

Q & A

Contact

Sabyasachee (Sabya) Mishra, Ph.D., P.E
Assistant Professor
Department of Civil Engineering
University of Memphis
E-mail: smishra3@memphis.edu